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EC2 SERVICEABILITY CHECK OF POST-TENSIONED ELEMENTS[1]

Bijan O Aalami[2]

This Technical Note explains the serviceability check of post-tensioned elements, with emphasis on floor systems, when using EC2 [EC2, 2004] code provisions. Where beneficial, the provisions are compared with those of ACI 318 [ACI 318, 2011]. The Technical Note is an expanded version of the information in the book on Design of Post-Tensioned Buildings [Aalami, 2014].

The provisions of the European Code EC2[3] are generally more complex than ACI 318 to navigate for design of a common post-tensioned floor system. The following presents a specific path through EC2 provisions, that when followed, will conclude with a code-compliant design.

The thrust of serviceability check of a floor system in EC2 – apart from deflection check – is crack control. In EC2-based designs, cracks are anticipated and permitted to form, as long as their impact on the durability of the structure, and their visual effects are acceptable. The design features an assumed "design crack width" for probable cracks under service loads. The values are generally between 0.1 to 0.3 mm (0.004 to 0.012 in.). The serviceability check is performed for both sustained "quasi-permanent" and total "frequent" load combinations.

A detailed numerical design based on the application of EC2 to a column supported two-way system is given in [Aalami, 2014]

The recommendations of EC2 for serviceability check are summarized in three flow charts marked A, B and C. The flow charts outline the sequence of steps that a designer needs to follow in order to satisfy the serviceability requirements of a post-tensioned element – in particular floor systems. The numerically marked paragraphs refer to the steps of the flow chart that bear the same number.

The relationships given use SI system of units (N, mm). The flow charts are placed at the end of this section.

(1) Determine Design Values

At this stage, it is assumed that the analysis of the floor system, including its post-tensioning is complete; design strips and design sections have been identified; the cross-sectional geometry of each design section is known; and the design actions for each section are calculated. The design actions generally include moments, shears and axial loads. The serviceability check reviews the status of each design section and determines whether the width of the

[2] Professor Emeritus, San Francisco State University; Principal, ADAPT Corporation; www.adaptsoft.com
[3] EN 1992-1-1:2004(E)),Section 7

probable cracks are within the acceptable range, and whether additional reinforcement beyond what is available at the section is necessary.

(2) Minimum Overall Reinforcement:
Each design section shall be checked to have a total amount of prestressed and non-prestressed reinforcement not less than a specified minimum amount, and not more than a specified maximum value. This step determines the minimum reinforcement associated with each design section. At this stage the layout and amount of prestressing at each design section are known. Also, a designer may have selected a certain amount of non-prestressed base-rebar, such as a top or bottom mesh, or a given amount of bars at specific locations. If the total amount of provided reinforcement is less than the minimum required, the shortfall is added. The minimum reinforcement depends both on the cross-sectional geometry of the design section and in the case of beams its moment capacity, if the beam is reinforced with unbonded tendons.

A. Based on Cross-Sectional Area: The minimum amount of reinforcement A_{smin}, is given in EC2 Section 9.2.1.1 for beams, and Section 9.3.1.1 for slabs. Both sections use the following relationship.

$$A_{smin} \geq \frac{0.26 b_t d f_{ctm}}{f_{yk}} \geq 0.0013 b_t d \qquad (\text{Exp}4.10.3(2)-1)$$

Where, in SI units

d = depth to the centroid of non-prestressed steel. The distance (d) refers to where non-prestressed steel is either located or would be positioned, where needed;
b_t = mean width of the tension zone;
f_{ctm} = mean axial tensile strength of concrete according to Table 3.1[4] of the code; and
f_{yk} = yield stress of non-prestressed reinforcement; f_{pk} is used in lieu of f_{yk}, where section is prestressed.

If both prestressed and non-prestressed steel are present in a design section, weighted average of their characteristic strengths is used. The intent of this provision is crack control arising from shrinkage and temperature changes.

The provided reinforcement to be compared with the minimum required is given by:

$$A_{sprov} = A_s + A_{ps} \times \frac{f_{pk}}{f_{yk}} \qquad (\text{Exp}4.10.3(2)-2)$$

The reason the area of prestressed steel (A_{ps}) is enhanced by the ratio given above is to indirectly recognize the precompression it provides. In theory, precompression alone could be adequate to mitigate shrinkage and temperature cracking. Otherwise, based on tensile stress that would develop due to shrinkage, the adjustment in area of prestressing steel would not have been justified.

[4] EN 1992-1-12004(E), Table 3.1

If the provided (A_{sprov}) reinforcement is less than the minimum determined (A_{smin}), increase the reinforcement to satisfy the minimum value.

B. Based on Moment Capacity: The total amount of prestressed and non-prestressed reinforcement at any section of a beam reinforced with unbonded tendons shall be adequate to develop a moment capacity at that section not less than 1.15 times the cracking moment computed for the same section[5].

Comment:
It is interesting to note that ACI 318 has a similar provision for the post-cracking strength of elements reinforced with "grouted" tendons, whereas in EC2 the requirement applies to elements reinforced with "unbonded" tendons.

(3) Maximum Overall Reinforcement:
Each design section is checked not to contain more than a maximum total reinforcement (A_{smax}), using the EC2 code Section 9.2.1.1 for beams and 9.3.1.1 for slabs. Both sections use the following relationship.

$$A_{smax} = 0.04 A_c \qquad \text{(Exp4.10.3(3) - 1)}$$

Where, A_c is the gross cross-sectional area of the design section.

If both prestressed and nonprestressed steel are present, weighted average of their characteristic strengths is used.

$$A_{sprov} = A_s + A_{ps} \times \frac{f_{pk}}{f_{yk}} \qquad \left(\text{Exp}4.10.3(3)-2\right)$$

If the provided steel (A_{sprov}) is more than the maximum allowable A_{smax}, the design has to be revised as indicated in item (8) of the flow chart.

(4) Frequent (Total) Load Combination[6]
Serviceability check is performed for both the frequent and quasi-permanent load combinations. We start with forming the frequent load combination by selecting the appropriate value of ψ, based on the occupancy of the floor system. See Section 4.10.1 for suggested values.

For residential and office buildings, the load combination is:

1.00 Selfweight + 1.00 DL + ψ LL + 1.00 PT

(5) Select the threshold stress values associated with "frequent" load combination for the materials used. These are:

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[5] EN 1992-1-12004(E) Section 9.2.1.1(4)
[6] Design Aids for Eurocode 2, part 1 [ENV 1992-1-1], Section 4.1

- Concrete
  - Maximum allowable compressive fiber stress[7] [0.60 f_{ck}].
  - Maximum allowable hypothetical tensile stress in concrete ($f_{ct,eff}$)[8]. Where calculated values exceed this threshold, crack control reinforcement may have to be added as noted farther in the flow chart.
- Nonprestressed reinforcement
  - The maximum allowable stress is (0.80 f_{yk})[9] .
- Prestressing steel
  - The maximum allowable stress under service condition is (0.75 f_{pk})[10] .

(6) Calculate the Hypothetical Extreme Fiber Stresses

In slab construction the local stress at the surface of concrete varies significantly from one point to the next. Stresses are high over the supports and drop rapidly with distance away from the support. In practice, rather than focusing on, or calculating stresses at a point, "hypothetical" stresses associated with selected "design sections" are calculated and used as values for crack control. Computed stress at a "point" bears no design significance for in-service response of a floor slab, since there is a poor correlation between computed local stresses and the actual values in prototype construction. Hypothetical stresses based on design sections of finite length are used to evaluate the likelihood of crack formation and their overall control. The selection of design sections and the computation of hypothetical stresses are outlined in Chapter 4 of reference [
Aalami, 2014].

(7) Compare Computed and Allowable Stress Values.

(7.1) Stress in prestressing tendons: This is to control the tendon stress in service not exceed the allowable value in (5). Through friction loss and long-term stress losses, this provision is generally satisfied, since tendon stress in service is generally about *0.60* f_{pk}. The provision is noted here for completeness, but it does not generally apply. It is automatically satisfied in common construction.

(7.2) Stress in non-prestressed reinforcement: This is a requirement, but in practice it is not generally carried out, since in common construction, the stress in non-prestressed reinforcement is typically much less than the code threshold. The provision is included for completeness, and unusual conditions.

(7.3) Concrete compression stress check: If the maximum hypothetical compressive stress in concrete is more than the allowable, the design of the section has to be revised. Selection of a larger section, or possibly reduction of prestressing is two of the possible remedial choices.

(8) Modify Design

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[7] EN 1992-1-1:2004(E), Section 7.2(2)
[8] EN 1992-1-1:2004(E), Section 7.1(2) and 7.3.2(4)
[9] EN 1992-1-1:2004(E), Section 7.2(5)
[10] EN 1992-1-1:2004(E), Section 7.2(5)

If the calculated stresses in the prestressing steel or non-prestressed steel or compressive stresses in concrete exceed the respective allowable values, the parameters of design have to be modified to lower the exceeding stresses. The structure has to be re-analyzed to determine the new design values, and stresses re-checked.

(9) Compare Hypothetical Concrete Tension Stress with Threshold Limit
The hypothetically calculated extreme fiber tension stress of the design section is compared with the threshold value stated in step 5. The outcome determines whether the section has to be checked for crack control, and if so, what measures have to be followed.

If the hypothetical stresses do not exceed the stated threshold, no crack control reinforcement will be needed for the "frequent" serviceability limit condition. For this case, the serviceability check will be continued following Flow Chart B that covers the "quasi permanent" load combination.

If the hypothetical tensile stresses exceed the stated threshold, probability of crack formation exists. Non-prestressed bonded reinforcement should be added to control the formation and width of probable cracks. Go to Flow Chart C.

(10) Flow chart C deals with the computation of the minimum non-prestressed bonded reinforcement (A_{scrack}) for crack control. Details are in the following steps

(11) The minimum area of bonded reinforcement for crack control (A_{scrack}) depends on the post-tensioning system used. Select "bonded" or "unbonded" system. As it is reflected further on, more reinforcement may be necessary than the minimum.

(12) Crack Control Reinforcement for Unbonded Systems (A_{scrack})
Since the hypothetical tension stress at the farthest fiber exceeded a lower allowable threshold, control of potential cracks becomes necessary for the affected design sections.

The requirement of minimum reinforcement for crack control is met by providing bonded reinforcement in the tensile zone of the concrete section. The minimum amount of the bonded reinforcement necessary under this provision will be increased, if crack width controls that are in the other steps of the flow chart conclude with additional reinforcement.

The requirement will be first expressed in its simplified format applicable to floor systems. Then, it will be presented in the EC2 code format for the general application.

$$A_{scrack} = \frac{N_c}{f_{yk}} \qquad \text{(Exp 4.10.3(12)-1)}$$

Where,

N_c = Tensile force over the tension zone of the concrete section (Fig. 4.10.3-1); and

f_{yk} = yield stress of the reinforcement used for crack control.

Note that f_{yk} is much higher than the value suggested for a similar condition in ACI 318. However, its recommendation in EC2 is justified, since the probability of crack formation, and crack width are addressed separately and accounted for – a step that is not detailed in ACI 318.

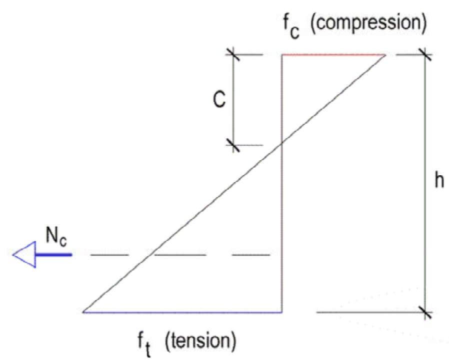

FIGURE 4.10.3-1 Computed Distribution of Stress over Concrete Section

The format of the preceding expression, as given in EC2 is more involved, since it is intended to cover the general scenario. The express is[11]:

$$A_{scrack} = \frac{k_c \times k \times f_{ct,eff} \times A_{ct}}{\sigma_s} \qquad \text{(Exp 4.10.3(12)-2)}$$

Where,

A_{ct} = area of concrete within tensile zone. The tensile zone is that part of the section which is calculated to be in tension just before formation of the first crack;

k = 1.0, for webs with h<= 300 mm or flanges with widths less than 300 mm;
= 0.65, for webs with h>= 800 mm or flanges with widths greater than 800 mm;
for other elements, linear interpolation is used.

k_c = determined based on the maximum fiber stresses as follows:

- For pure tension k_c = 1.0
- For bending or bending combined with axial forces:

[11] EN 1992-1-1:2004(E), Section 7.3.2(2)

$$k_c = 0.4\left[1 - \frac{\sigma_c}{k_1\left(h/h^*\right)f_{ct,eff}}\right] \le 1 \qquad \text{(Exp 4.10.3(12)-3)}$$

For flanges of T-sections:

$$k_c = 0.9\frac{F_{cr}}{A_{ct} \times f_{ct,eff}} \ge 0.5 \qquad \text{(Exp 4.10.3(12)-4)}$$

Where,

$$\sigma_c = \frac{N_{ED}}{bh}\text{; average precompression} \qquad \text{(Exp 4.10.3(12)-5)}$$

N_{ED} = axial force at the serviceability limit state;

h^* = h for h <1.0 m

= 1.0m for h ≥1.0 m

k_1 = 1.5 if N_{ED} is a compressive force

$$= \frac{2h^*}{3h} \quad \text{if } N_{ED} \text{is a tensile force}$$

F_{cr} = absolute value of the tensile force within the flange due to the cracking moment calculated with $f_{ct,eff}$;

$f_{ct,eff}$ = tensile strength of concrete at time of crack formation, f_{ctm}, but not less than 3 MPa[12];

f_{ctm} = mean axial tensile strength according to Table 3.1 of the EC2; and

σ_s = may be taken as yield stress of non-prestressed steel, f_{cyk}.

(13) Crack Control Reinforcement for Bonded Systems (A_{scrack}).

Since the hypothetical tension stress at the farthest fiber exceeded a lower allowable threshold, control of potential cracks becomes necessary for the affected design sections.

For bonded systems, the required minimum area of reinforcement is the same as described for the unbonded systems, with the difference that the available area of grouted tendons will be accounted for their contribution to crack control. In this respect, EC2 specifies a maximum spacing (300 mm[13]) between the reinforcement that is considered to be effective for crack control. For column-supported slab construction, this is viewed to serve the same objective as in ACI 318 provision for the spacing over the support intended for crack control and ductility – ACI 318[14] specifies not more than 12 " (305 mm). For the common case of column supported floor slabs, the EC2 provision translates to the following:

[12] Design Aids for Eurocode 2, Part 1[ENV 1992-1.1], page 76
[13] EC2[EN 1992-1-1:2004(E)], Section 7.3.2(3)
[14] Give aci bare spacing

$$A_{scrack} = \frac{N_c}{f_{yk}} - A_{ps} \qquad \text{(Exp 4.10.3(13)-1)}$$

Where,

A_{ps} = Cross-sectional area of bonded tendons deemed to be available at location of probable crack formation. For column supported floor construction this is considered to be the width of column support extended on each side by 1.5 times the element thickness.

As it is outlined below in the detailed expression, EC2 has a restriction on the fraction of the cross-sectional area of the bonded tendons that can be included in the preceding relationship. However, the restriction does not apply to floor systems, on the premise that the effective stress in tendons under service is generally between 60 to 65 % of tendon's ultimate strength, or less. And, the difference between the effective stress of tendon in service, and tendon's ultimate strength is in excess of the yield stress of non-prestressed reinforcement used for crack control. In other words, stress in a tendon in service can be raised by the amount of the yield strength of an adjacent non-prestressed reinforcement, without failing the tendon.

For bonded systems, the minimum area of reinforcement (A_{scrack}) required for crack control is given by the following[15] :

$$A_{scrack} = \frac{k_c k f_{ct,eff} A_{ct} - \xi_1 A_p^{'} \Delta\sigma_p}{\sigma_s} \qquad \text{(Exp 4.10.3(13)-2)}$$

Where, the definitions of the new symbols are:

ξ_1 = the adjusted ratio of bond strength taking into account the different diameters of prestressing and reinforcing steel.

$$= \sqrt{\xi \frac{\phi_s}{\phi_p}} \qquad \text{(Exp 4.10.3(13)-3)}$$

ξ = ratio of bond strength of prestressing and steel according to Table 6.2[16] of EC2;
Φ_s = largest bar diameter of reinforcing steel;
Φ_p = equivalent diameter of tendon according to EC2 6.8.2;
= $1.6\sqrt{A_p}$ for bundles, where A_p is the area of prestressing tendon or tendons
= $1.75\ \Phi_{wire}$ for single 7 wire strands where Φ_{wire} is the wire diameter

If only prestressing steel is used to control cracking, $\xi_1 = \sqrt{\xi}$

$A_{p'}$ = area of tendons within $A_{c,eff}$;

[15] EC2[EN 1992-1-1:2004(E)], Section 7.3.2(3)
[16] EN 1992-1-1:2004(E), Sections 7.3.2(3) and 6.8.2

$A_{c,eff}$ = effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth $h_{c,ef}$;
$h_{c,ef}$ = lesser of $2.5(h-d)$, $(h-x)/3$ or $h/2$;
h = depth of the element; and
$\Delta\sigma_p$ = stress variation in prestressing tendons from the state of zero strain of the concrete at the same level.

(14) Select Allowable Crack Width

The allowable crack width for each floor system depends on the anticipated exposure of the floor to corrosive elements, and/or the aesthetic impact of probable cracks. The exposure classifications[17] and the recommended values are given in Table 7.1N of the code. The values range from 0.2 to 0.4 mm. For elements reinforced with unbonded tendons, the most common selection is 0.3 mm. For elements reinforced with bonded systems, the suggested value for most exposures is 0.2 mm.

(15) Calculate "Computed" Crack Width

Using the procedure described below[18], and the provision of bonded non-prestressed reinforcement from the previous steps, the serviceability check continues with the computation of the probable crack width (w_k) for each design section. The calculation of the probable crack width (w_k) is explained below. A numerical example of it is given in Chapter 6 of reference [Aalami, 2014].

Computed probable crack width,

$$w_k = s_{r,max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right) \qquad \text{(Exp 4.10.3(15)-1)}$$

Where,

$s_{r,max}$ = maximum crack spacing;
ε_{sm} = mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening; and
ε_{cm} = mean strain in the concrete between cracks.

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \geq 0.6\frac{\sigma_s}{E_s} \qquad \text{(Exp 4.10.3(15)-2)}$$

Where,

σ_s = the stress in the tension reinforcement calculated on the basis of a cracked section [f_{yk}];
α_e = E_s/E_{cm}

$$\rho_{p,eff} = \frac{\left(A_s + \xi_1^{\,2} \times A_p'\right)}{A_{c,eff}} \qquad \text{(Exp4.10.3(15) –3)}$$

[17] EN 1992-1-1:2004(E), Section 4.2, Table 4.1
[18] EN 1992-1-1:2004(E), Section 7.3.4.

$A_{p'}$ = area of tendons within $A_{c,eff}$
$A_{c,eff}$ = effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth $h_{c,ef}$;
$h_{c,ef}$ = lesser of $2.5(h\text{-}d)$, $(h\text{-}x)/3$ or $h/2$;
h = depth of the element;
x = depth of neutral axis from the compression fiber;

$$\xi_1 = \sqrt{\xi \frac{\phi_s}{\phi_p}} \qquad \left(\text{Exp 4.10.3}\left(12\right)\text{-5}\right)$$

ξ = ratio of bond strength of prestressing and steel according to Table 6.2;
Φ_s = largest bar diameter of reinforcing steel;
Φ_p = equivalent diameter of tendon according to 6.8.2;
k_t = factor dependent on the duration of the load;
= 0.6 for short term loading
= 0.4 for long term loading
E_s = modulus of elasticity of steel; and
$s_{r,max} = 1.3(h\text{-}x)$ (Exp 4.10.3(15)-4)

(16) Limit Crack Width
If the computed probable crack width from the previous step exceeds the value selected for design, the code provides two remedial options. Either add reinforcement using the following relationship to limit the probable crack width (w_k), or select non-prestressed bar diameter and spacing according to Table 7.2 N or 7.3 N. Adding reinforcement will increase $\rho_{p,eff}$ in the following.

$$w_k = s_{r,max}\left(\frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}\left(1+\alpha_e \rho_{p,eff}\right)}{E_s}\right) \qquad \left(\text{Exp4.10.3}\left(16\right)-1\right)$$

Where,

$$\rho_{p,eff} = \frac{\left(A_s + \xi_1^{\,2} \times A_p{'}\right)}{A_{c,eff}} \qquad \text{(Exp 4.10.3(15)-3)}$$

(17) Quasi-Permanent (sustained) Load Combination[19]
The general load combination is:

1.00 Selfweight + 1.00 DL + ψ LL + 1.00 PT

[19] Design Aids for Eurocode 2, part 1 [ENV 1992-1-1], Section 4.1

The recommended values for ψ are given in Section 4.10.1. For common residential and commercial buildings, the recommended value is 0.3.

(18) Select Stress Thresholds

Stress limitations used for the "quasi-permanent" load combination are as follows:

- Concrete
  - Maximum threshold for compressive stress is ($0.45f_{ck}$)[20] .
  - Maximum threshold for hypothetical tensile stress is ($f_{ct,eff}$)[21].
- Nonprestressed reinforcement
  - No additional stress check is required.
- Prestressing steel
  - No additional stress check is required.

(19) Calculate the Hypothetical Concrete Stresses

Extreme fiber concrete stresses for each design section are calculated using the same procedure outlined in the main flow chart.

(20) Modify Calculated Deflection due to Creep

If the hypothetical farthest fiber compressive stress of concrete in a design section exceeds the threshold for this load combination, the structure is anticipated to undergo greater deformation due to increased creep in concrete. The code recommends using a non-linear procedure for the creep component of the deformation in the structure when calculating the design deflections.

REFERENCES

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ACI 318-11, (2011), "*Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary,*" American Concrete Institute, Farmington Hill, MI 48331, www.concrete.org, 503 pp.

European Code EC2, (2004), "*Eurocode 2: Design of Concrete Structures – Part 1-1 General rules and rules for buildings,*" European Standard EN 1992-1-1:2004.

[20] EN 1992-1-1:2004(E), Section 7.2(3)
[21] EN 1992-1-1:2004(E), Sections 7.1(2) and 7.3.2(4)

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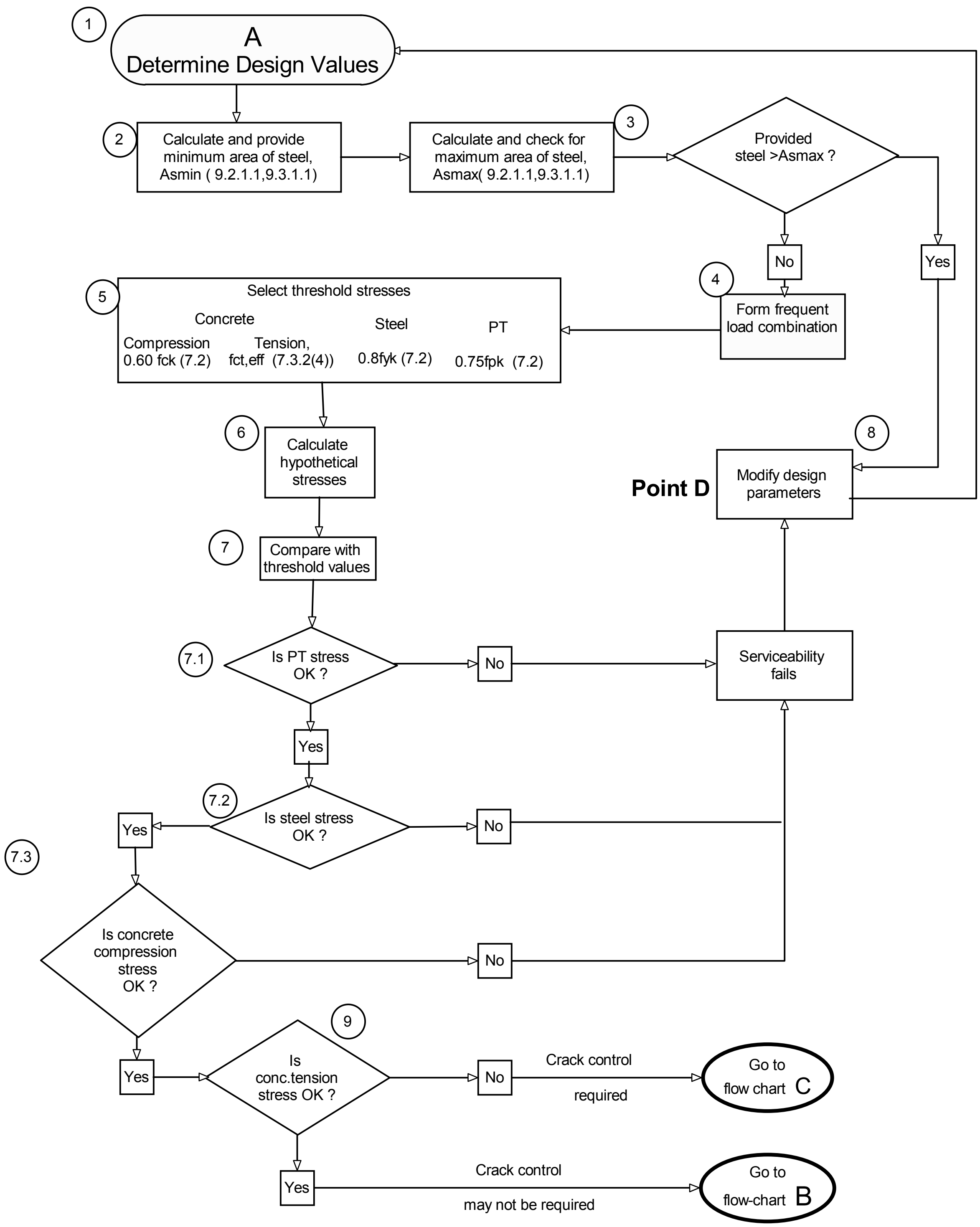

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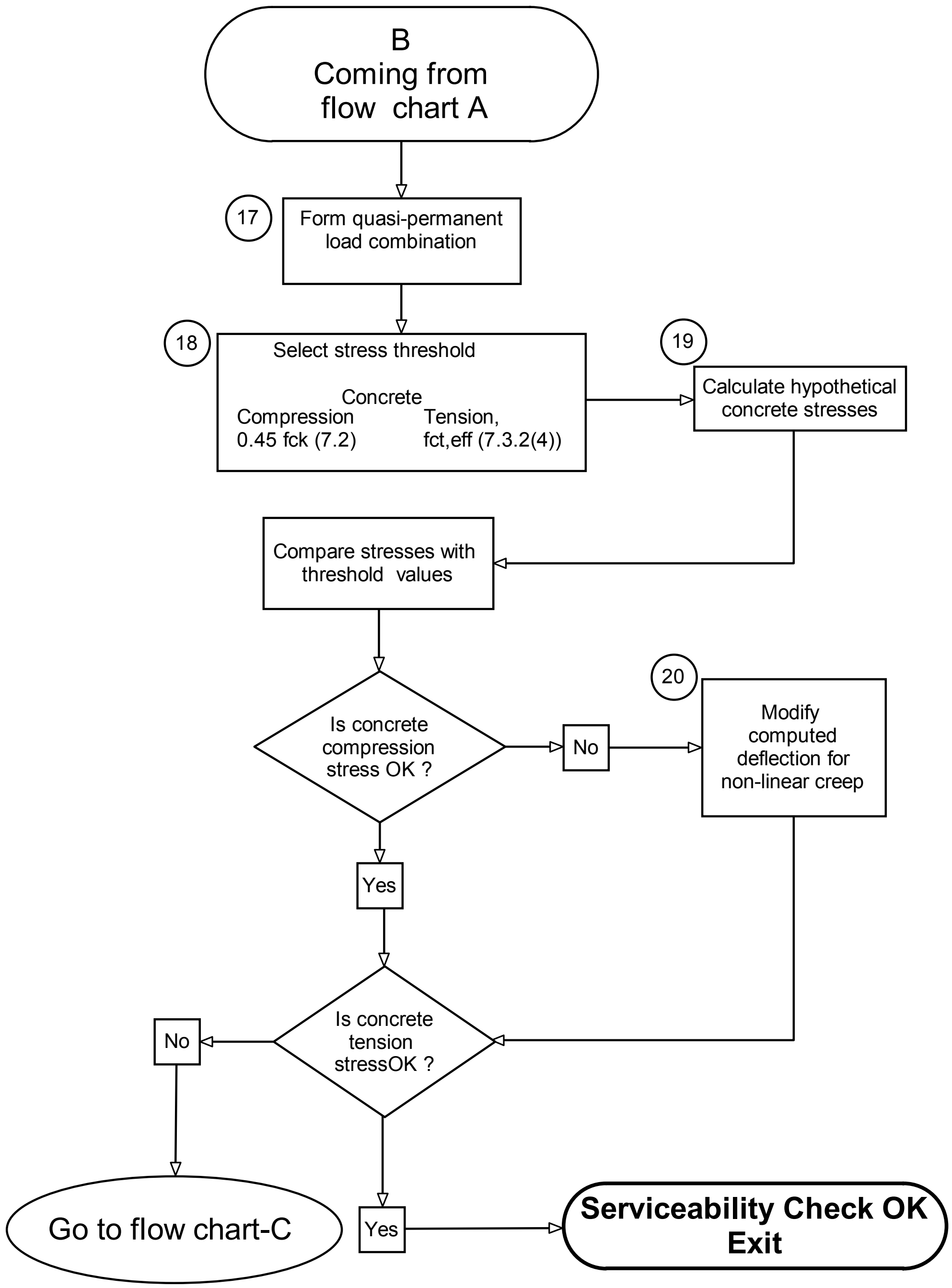

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